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Plastic flow of persistent currents in strongly interacting systems: manifestations of the few-channel characteristics

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Abstract
The persistent-current characteristics of two-dimensional strongly interacting systems are investigated. The response to an external magnetic field, the sensitivity to addition of a single scatterer and the noise autocorrelation in the strongly interacting regime are different to those in the non-interacting diffusive regime. The response in the strongly interacting regime is consistent with a persistent current carried by a series of one-dimensional channels.

1. Introduction

The behaviour of interacting electrons in random potentials has recently attracted much interest because of its association with the experimentally observed temperature dependence of the conductivity for 2D (two-dimensional) low-density systems known as the 2D MIT (metal–insulator transition) [1]. This interest has led to many recent numerical studies of the interplay between interactions and disorder for small two-dimensional clusters [2–13]. One interesting behaviour which has emerged is the change in the local distribution of the persistent current for low densities of electrons [14, 15]. It turns out that while in the high-density (weak-interaction) regime the local persistent current is diffusive, once the density is reduced (or in other words the e–e (electron–electron) interactions become stronger) the local current changes its character. The local persistent current in the low-density regime flows in several well defined channels, mainly in the direction perpendicular to the applied flux, with no closed loops [7, 15, 16]. These channels usually appear at the boundaries between domains of Wigner crystals; the scenario is reminiscent of plastic flow of dislocations in crystals. There have even been recent claims that the change in the flow patterns of the persistent current is associated with a new quantum phase in the system, and is associated with the 2D MIT [7]. We believe that although the question of whether the transition of the persistent-current flow is a real phase transition or just an intermediate region in phase space has not yet been decided, and although this behaviour is not directly relevant to the 2D MIT [17], it is nevertheless important to consider its consequences.

Since it is very hard (if not impossible) to measure the local persistent current, it is of interest to find a method for seeing this crossover in the global properties of the persistent current. In this paper we wish to address some of the global characteristics of the persistent current in the low-density regime. We shall demonstrate that the response of the system to the application of an external perturbation such as an additional scatterer or a magnetic field depends on the strength of the e–e interactions. The response in the low-density (strong-interaction) regime is very different from the response in the high-density (weak-interaction) one. The response of the persistent current in the low-density regime is consistent with the response expected from a system in which the current flows in several quasi-one-dimensional channels through the system. This indicates that in the low-density regime one should expect the response of the two-dimensional systems to follow a quasi-one-dimensional behaviour for various experimental measurements, such as sensitivity to movement of a single impurity, $1/f$ noise and response to an external magnetic field. These fingerprints of the low-density behaviour may turn out to be useful in a detailed study of these systems.

2. Model

We study the behaviour of an interacting 2D cylinder of circumference L_x and height L_y threaded by a flux ϕ and by a perpendicular magnetic field B (see figure 1). This system is known to show a large enhancement of the persistent current in the diffusive regime [18]. The Hamiltonian is given by

$$H = \sum_{k,j} \epsilon_{k,j} a^\dagger_{k,j} a_{k,j} - \mathrm{i} \sum_{k,j} (J^x_{k,j} + J^y_{k,j}) + U \sum_{\{k,j\}>\{l,p\}} \frac{(a^\dagger_{k,j} a_{k,j} - K)(a^\dagger_{l,p} a_{l,p} - K)}{|\vec{r}_{k,j} - \vec{r}_{l,p}|} \tag{1}$$

where

$$\begin{aligned} J^x_{k,j} &= -\mathrm{i}V \exp(\mathrm{i}\Phi s/L_x + \mathrm{i}\theta k) a^\dagger_{k,j+1} a_{k,j} - \mathrm{h.c} \\ J^y_{k,j} &= -\mathrm{i}V a^\dagger_{k+1,j} a_{k,j} - \mathrm{h.c} \end{aligned} \tag{2}$$

and $a^\dagger_{k,j}$ is the fermionic creation operator, $\epsilon_{k,j}$ is the energy of a site (k, j), which is chosen randomly between $-W/2$ and $W/2$ with uniform distribution, V is a constant hopping matrix element and s is the lattice constant. The magnetic field is represented in the Landau gauge [19] $\theta = 2\pi Bs^2/\phi_0$ (where ϕ_0 is the quantum flux unit) and $\Phi = 2\pi\phi/\phi_0$. The distance $|\vec{r}_{k,j} - \vec{r}_{l,p}| = ((k-l)^2 + \min\{(j-p)^2, (L_x/s - |j-p|)^2\})^{1/2}$ and a positive background charge, K, equal to the electron density was added. The interaction term represents a Coulomb interaction between electrons confined to a 2D cylinder embedded in a 3D space with $U = e^2/s$.

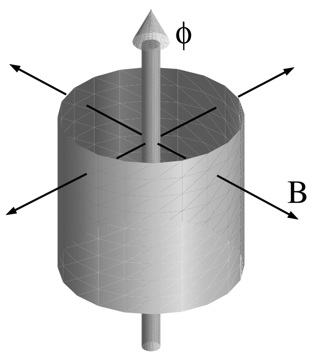

Figure 1. The 2D cylinder threaded by a flux ϕ and subjected to a perpendicular magnetic field B.

We consider a 4×4 (and a 5×5) lattice with $m = 16$ ($m = 25$) sites and $n = 8$ ($n = 4$) electrons. The many-particle Hamiltonian is represented by a $\binom{m}{n} \times \binom{m}{n}$ ($12\,870 \times 12\,870$ and $12\,650 \times 12\,650$) matrix, which is exactly diagonalized. The many-particle ground state $|\Psi(\Phi)\rangle$ is calculated for 3000 different realizations of disorder in the diffusive regime ($W = 8V$ [18]) for several values of the interaction U. The local persistent current

$$I^a_{k,j}(\Phi) = \langle\Psi(\Phi)|J^a_{k,j}|\Psi(\Phi)\rangle \tag{3}$$

(where $a = x, y$) is calculated for each realization.

For typical 2DEG devices used in the recent 2D MIT experiments the density is rather low (for example, usually in GaAs $n < 10^{11}$ cm^{-2}) which corresponds to a ratio between the Coulomb and Fermi energy $r_s = 1/\sqrt{\pi n} a_B > 3$, where a_B is the Bohr radius. In this region the simple RPA (random-phase approximation) no longer holds, and correlations play an important role as can be seen from the behaviour of the persistent current [18, 20] and Coulomb blockade phenomena in quantum dots [21–24]. For a tight-binding system $r_s \sim \sqrt{m/4\pi n}(U/V)s$, which corresponds to $r_s \sim 4$ at $U = 10V$ for the 4×4 system and $r_s \sim 7$ at $U = 10V$ for the 5×5 system. For these values of interaction it is known from previous studies [7, 14–16] that the local persistent current changes its characteristics and is no longer diffusive. Strong density correlations appear, for both the cylinder [14, 15] and torus [7, 16] geometries. The electrons form on short length scales a Wigner crystal with dislocations, as can be seen in figure 2 of reference [15]. This is the region which we are interested in studying in more detail.

3. Perpendicular magnetic field

The response to a perpendicular magnetic field is very different for 2D and quasi-1D systems. While 2D systems are sensitive to weak perpendicular magnetic fields, quasi-1D systems are not. This stems from the fact that the sensitivity of a mesoscopic system to a weak perpendicular magnetic field is the result of the fact that the interference of closed loops depends on the magnetic flux that they encompass [25]. In a 2D system there are many closed loops and therefore the persistent current (or any other physical property such as conductance, or local density) will be sensitive to the perpendicular magnetic field. On the other hand, in quasi-1D systems there are no closed loops and therefore one does not expect a quasi-1D system to be sensitive to weak perpendicular magnetic fields. Therefore, sensitivity to a perpendicular magnetic field should give a very clear indication of the dimensionality of the system.

In figure 2 (figure 3) we plot the total current

$$I(\Phi = \pi/2) = \sum_k I^x_{k,j}(\Phi = \pi/2) \tag{4}$$

for 4×4 (5×5) systems. Of course, the current does not depend on which column j the summation is performed on. The average current $\langle I(\Phi = \pi/2)\rangle$ (where $\langle\cdots\rangle$ denotes averaging over different realizations of disorder) and the root mean square value of the current

$$(\langle\delta^2 I(\Phi = \pi/2)\rangle)^{1/2} = (\langle(I(\Phi = \pi/2))^2\rangle - \langle I(\Phi = \pi/2)\rangle^2)^{1/2} \tag{5}$$

as a function of the interaction strength are presented in figure 2(a) (figure 3(a)) both in the absence and in the presence of a magnetic field. The usual dependence of the current on the interaction strength is seen [18]. The average current is enhanced by medium-strength interaction, but as the interaction becomes stronger a drop in the average current as result of the Mott–Hubbard transition is seen for the 4×4 systems. The new information in this figure pertains to the response to the perpendicular magnetic field. It is clearly seen that the influence of the perpendicular magnetic field is much less significant for higher values of interaction.

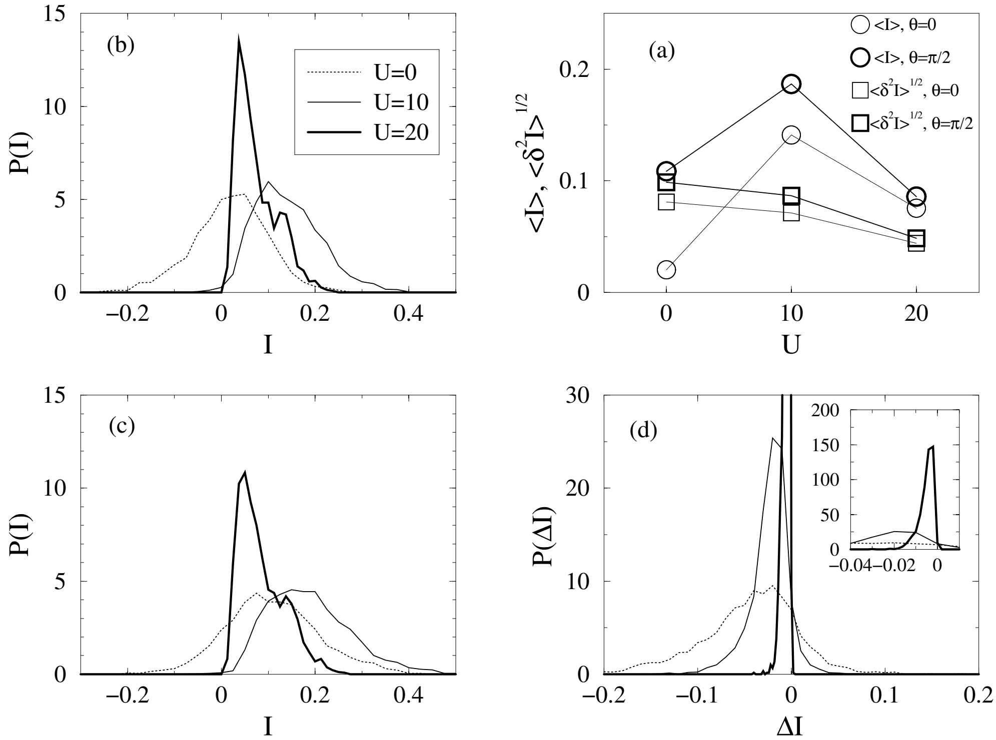

Figure 2. The persistent current of a cylinder at a threading flux $\Phi = \pi/2$, in the absence ($B = 0$) and presence ($B = 0.25\phi_0/s^2$) of a perpendicular magnetic field for 4×4 samples and different strengths of the electron–electron interactions, U. (a) The average and root mean square values of the persistent current. (b) The distribution of the persistent current for $B = 0$. (c) The distribution of the persistent current for $B = 0.25\phi_0/s^2$. (d) The distribution of $\Delta I = 0.5(I(B = 0) - I(B = 0.25\phi_0/s^2)$. Inset: an enlargement of the region around $\Delta I = 0$.

While there exists a significant difference between $\langle I(\Phi = \pi/2)\rangle$ in the absence and presence of a magnetic field for $U = 0$, there is almost no difference for $U = 20V$.

The reduced sensitivity to a perpendicular magnetic field is seen even more clearly by studying the distribution of the total current. In figures 2(b), 2(c) (figures 3(b), 3(c)) the distributions of the total current $I(\Phi = \pi/2)$ in the absence and in the presence of a magnetic field are presented. While for $U = 0$ the magnetic field shows a considerable influence on the distribution (since the magnetic field strongly influences the persistent current for 2D systems [19]), for stronger interactions there is a smaller shift in the distributions. Plotting the distribution of the current change for each sample $P(\Delta I(\Phi = \pi/2))$, where

$$\Delta I(\Phi = \pi/2) = \frac{1}{2}(I(\Phi = \pi/2, B = 0) - I(\Phi = \pi/2, B \neq 0)) \qquad (6)$$

one can clearly see, figure 2(d) (figure 3(d)), that there is a broad distribution of changes in the current due to the magnetic field at $U = 0$ as can be expected from a diffusive system. On the other hand, as the interaction strength is enhanced the distribution narrows considerably, corresponding to a weak influence of the magnetic field.

Another interesting signature of the crossover from a diffusive 2D behaviour for weak interactions to a quasi-1D behaviour for strong interactions is the behaviour of $\langle(I(\Phi = 0, B \neq 0))^2\rangle$. In the absence of a magnetic field there is of course no persistent current

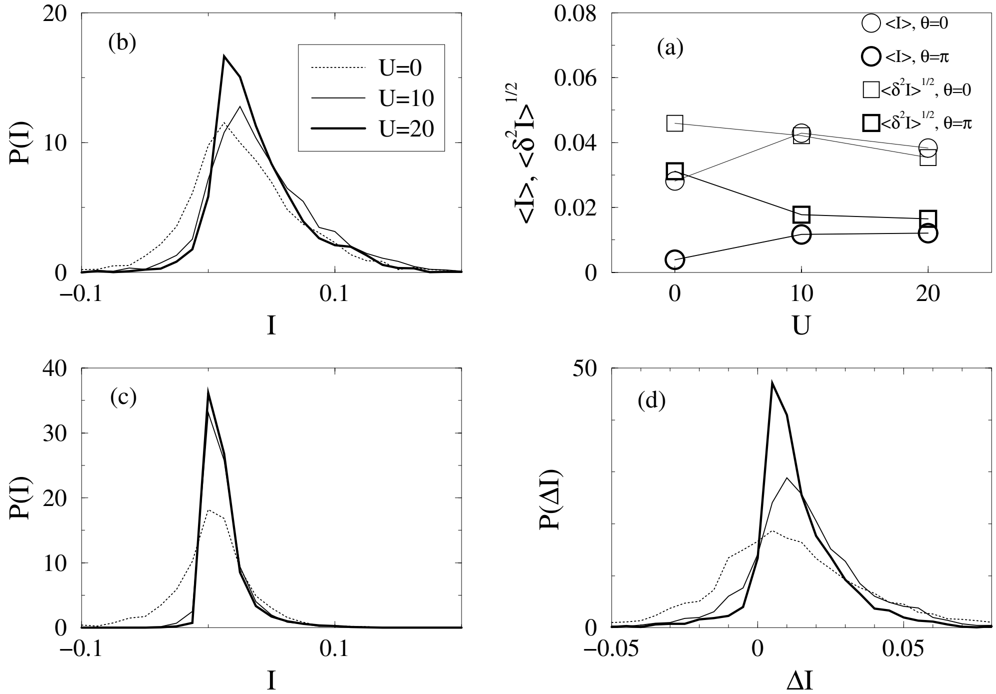

Figure 3. The persistent current of a cylinder at a threading flux $\Phi = \pi/2$, in the absence ($B = 0$) and presence ($B = 0.5\phi_0/s^2$) of a perpendicular magnetic field for 5×5 samples and different strengths of the electron–electron interactions, U. (a) The average and root mean square values of the persistent current. (b) The distribution of the persistent current for $B = 0$. (c) The distribution of the persistent current for $B = 0.5\phi_0/s^2$. (d) The distribution of $\Delta I = 0.5(I(B = 0) - I(B = 0.5\phi_0/s^2)$. Inset: an enlargement of the region around $\Delta I = 0$.

for zero flux. Nevertheless, once a magnetic field is applied, the persistent current even in the absence of an external flux does not vanish [19]. The average current $\langle I(\Phi = 0, B \neq 0)\rangle = 0$, since on average there is no preferential direction for the current, but for a particular realization of disorder there exists a non-vanishing current driven by the magnetic field. Thus, we expect $\langle(I(\Phi = 0, B \neq 0))^2\rangle > 0$ for a 2D system. On the other hand, a 1D system does not encompass any flux due to the magnetic field, resulting in $\langle(I(\Phi = 0, B \neq 0))^2\rangle = 0$. Indeed in figure 4 it can be seen that $\langle(I(\Phi = 0, B \neq 0))^2\rangle$ tends to zero as the interaction increases.

4. Sensitivity to a single impurity

Another difference between 2D systems and quasi-1D systems is as regards their sensitivity to the motion of a single scatterer. In a diffusive 2D system all electron paths have a finite probability of crossing a specific impurity [26, 27], and therefore changing the position of a single scatterer will influence the outcome for the persistent current flowing in the system. On the other hand, in a quasi-1D situation, the motion of an impurity will influence only the persistent current carried by a channel close to it, and if the impurity happens not to be close to any channel it will have no influence at all. While in the diffusive regime the addition of a strong impurity to the system may either enhance or decrease the persistent current, in the strongly interacting case an addition of a strong impurity will either block the channel or have almost no influence at all, resulting only in a decrease of the current.

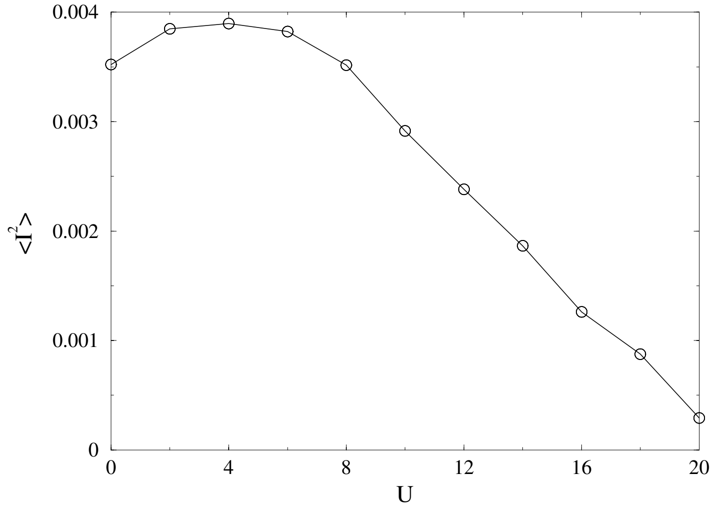

Figure 4. The average of the squared persistent current at a threading flux $\Phi = 0$ and a finite perpendicular magnetic field $B = 0.25\phi_0/s^2$ as function of U for 4×4 samples.

A strong impurity is introduced into the system by setting the hopping matrix element $V = 0$ for all the elements that connect a particular site to its neighbours. Thus, the current through the site is blocked while an electron may still reside on the site which mimics the behaviour of a local perturbation. We chose not to change the on-site energy of the site in order to avoid changes in the current resulting from a rearrangement of the Wigner crystal due to the addition of a new strong pinning centre. For small systems, such a rearrangement will completely change the current, since the correlation length of the Wigner crystal is larger than the sample length, resulting in effectively a new sample. On the other hand, for larger systems, the rearrangement of the crystal will be local, resulting in an effectively larger but still local impurity. The local persistent current is then calculated for the same realizations of disorder in the absence and in the presence of the additional impurity.

First let us consider the typical change in the persistent current due to the addition (or change in the position) of a scatterer:

$$\Delta^2 I \equiv \frac{1}{2}\,[I - I_1]^2 \tag{7}$$

where I (I_1) is the persistent current prior to (after) the addition of the scatterer. This is analogous to the definition of the typical change in the conductance due to the addition of a scatterer [26, 27]. In the non-interacting limit ($U = 0$) the average of this quantity was calculated using a diagrammatic perturbation theory leading to [28]

$$\langle \Delta^2 I \rangle \propto \langle I^2 \rangle \left(1 - \exp\left(-C\frac{L}{\ell}\sqrt{\frac{1}{m}}\right)\right) \tag{8}$$

where $L = \max\{L_x, L_y\}$, ℓ is the mean free path and C is a numerical constant of order one. The persistent current is exponentially sensitive to the addition of a strong impurity. On the other hand, for strong interactions the current flows through N independent channels. In this case one may express the total persistent current as an incoherent sum of the currents through these channels, i.e.,

$$I \sim \sum_{j=1}^{N} \iota_j \tag{9}$$

where $\imath_j$ is the persistent current through the jth channel. In such a case, the addition of a single impurity will change only the current through the kth channel on which it happens to fall. Thus,

$$I - I_1 = \sum_j (\imath_j - (\imath_j)_1) \sim \imath_k - (\imath_k)_1 \tag{10}$$

and in the case where the additional impurity is strong enough to block the current through the channel (i.e., $(\imath_k)_1 = 0$):

$$\langle \Delta^2 I \rangle \sim \frac{1}{2} \langle \imath_k^2 \rangle. \tag{11}$$

In order to estimate the influence of the interactions on the typical change we should compare the non-interacting typical change, $\langle I^2 \rangle (1 - \exp(-Cs/\ell))$ (assuming $m \sim (L/s)^2$), to $\langle \imath_k^2 \rangle$. Since typically $\imath_k \sim I/N$, we expect the typical change in the strongly interacting regime to be much smaller than in the non-interacting regime.

The numerical evaluation of the change in the persistent current due to the introduction of a strong scatterer is presented in figures 5, 6 and 7. As expected, the average and typical currents do not show very much change due to the introduction of the scatterer (see figures 5(a) and 6(a)). Nor does the distribution change much as can be seen in figures 5(b), 5(c)

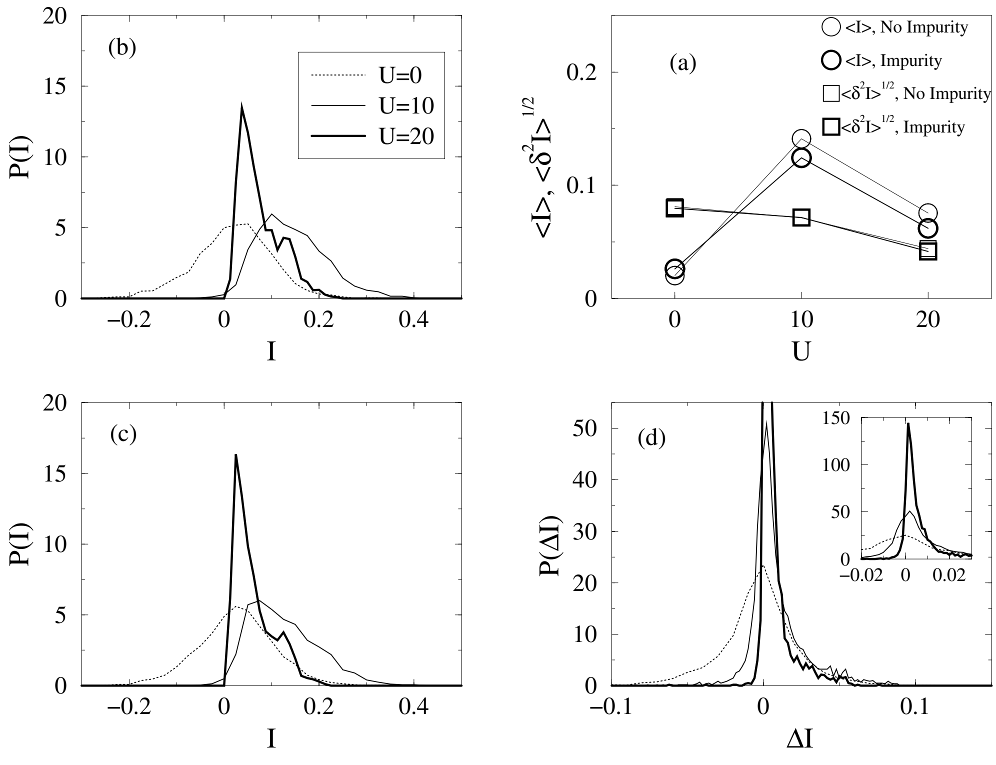

Figure 5. The persistent current of a cylinder at a threading flux $\Phi = \pi/2$, in the absence and presence of an additional strong impurity for 4×4 samples and different strengths of the electron–electron interactions, U. (a) The average and root mean square values of the persistent current. (b) The distribution of the persistent current with no additional impurity. (c) The distribution of the persistent current in the presence of an additional impurity. (d) The distribution of $\Delta I = 0.5(I - I_1)$. Inset: an enlargement of the region around $\Delta I = 0$.

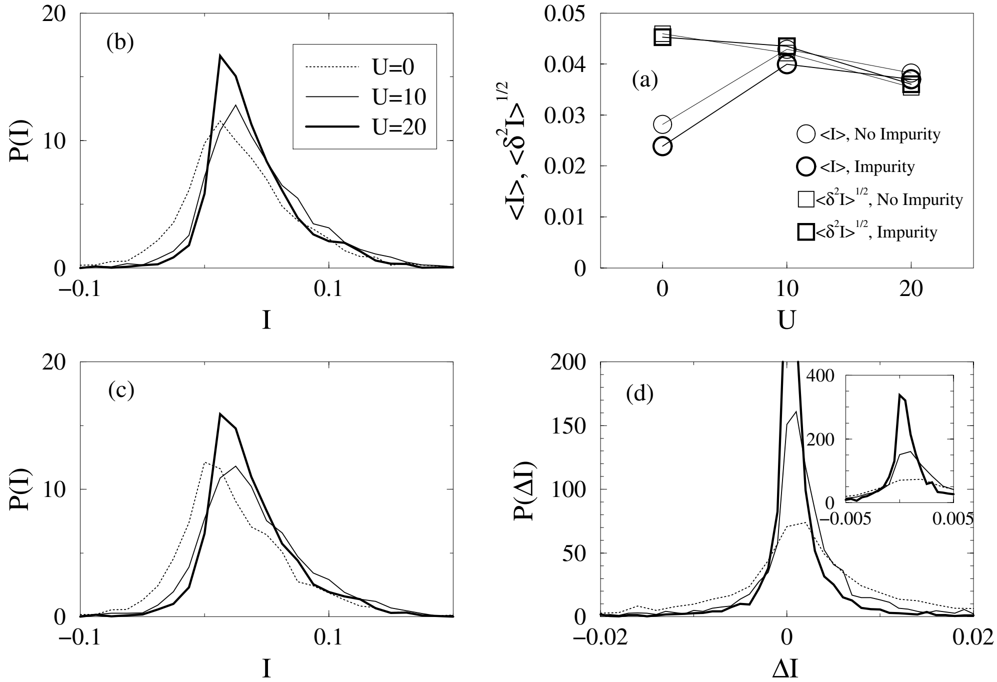

Figure 6. The persistent current of a cylinder at a threading flux $\Phi = \pi/2$, in the absence and presence of an additional strong impurity for 5×5 samples and different strengths of the electron–electron interactions, U. (a) The average and root mean square values of the persistent current. (b) The distribution of the persistent current with no additional impurity. (c) The distribution of the persistent current in the presence of an additional impurity. (d) The distribution of $\Delta I = 0.5(I - I_1)$. Inset: an enlargement of the region around $\Delta I = 0$.

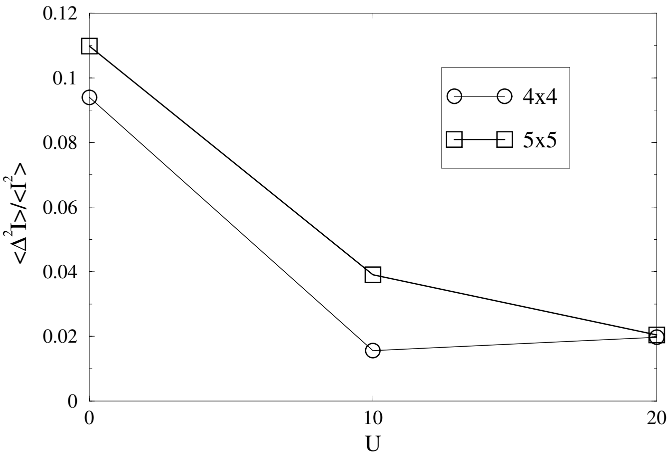

Figure 7. The typical change in the current $\langle \Delta^2 I\rangle / \langle I^2 \rangle$ as a function of the interaction strength, U.

and 6(b), 6(c). On the other hand, the change in the current of a sample due to the additional scatterer undergoes a change in its characteristics due to interactions. In the non-interacting case, the additional scatterer may increase or decrease the current in the sample with more or less equal probability (see figures 5(d) and 6(d)), while in the strong-interaction regime the current is either unchanged or suppressed. This is a clear manifestation of the crossover from the diffusive behaviour to a quasi-1D channel behaviour discussed above.

The typical change in the current as a function of the interaction strength depicted in figure 7 shows a strong suppression compared to the non-interacting value. At the limit of $U \to \infty$,

$$\frac{\langle \Delta^2 I \rangle}{\langle I^2 \rangle} \sim \langle \iota_k^2 \rangle \Big/ \left[2 \left(\sum_{j=1}^{N} \iota_j \right)^2 \right] \sim \frac{1}{N^2}. \tag{12}$$

For small systems (where N is small), $\langle \Delta^2 I \rangle / \langle I^2 \rangle$ is finite even for strong interactions, while for large systems it tends to zero.

5. Discussion

We have shown that different aspects of the behaviour of the persistent current as a function of the strength of the interaction (such as the response to a perpendicular magnetic field and the response to an additional scatterer) may be understood as the results of the change in the nature of the local current flow. As mentioned in the introduction, we are interested in finding experimentally accessible features of the current which signal the transition in the local current flow. In this respect both the magnetic field measurements and the addition of a single impurity to the sample are not ideal. It is hard in a real experimental geometry to differentiate between the perpendicular magnetic field and the flux, while its very hard to control the motion of scatterers within the sample, although a local potential perturbation may be introduced using an external gate. Fortunately, the noise properties of the persistent current seem to suggest an experimentally accessible way to study the local currents.

The noise autocorrelation of the persistent current is defined as

$$S_I(t) = \langle \delta I(t_0)\, \delta I(t_0 + t) \rangle \tag{13}$$

where $I(t)$ is the persistent current at specific time t. The power spectrum is given by

$$S_I(\omega) = \int S_I(t) \cos(\omega t)\, \mathrm{d}t. \tag{14}$$

Low-temperature noise in mesoscopic systems is usually ascribed to thermal motion of impurities. The thermal motion of the impurities depends on the material properties of the device and (to first order) not on the measured quantity. Thus, any coherent measurement, including that of the persistent current, will be affected by the thermal motion of impurities. By analogy with the noise spectrum of the voltage in disordered metals [29], we expect the noise spectrum of the persistent current to follow the canonical $1/f$ noise spectrum, i.e.,

$$S_I(\omega) = \frac{2\pi \alpha \langle I^2 \rangle}{m\omega} \tag{15}$$

where α is a material-dependent dimensionless parameter (which is known as the Hooge parameter [29]). Assuming that the noise in the persistent current originates from thermally activated motion of the impurities, one can estimate

$$\alpha = \frac{m(\langle \Delta I^2 \rangle / \langle I^2 \rangle)}{\ln(\omega_{max}/\omega_{min})} \tag{16}$$

where $\omega_{max\,(min)} \propto \exp(-E_{min\,(max)}/k_B T)$, $E_{min\,(max)}$ is the minimum (maximum) activation energy of an impurity. Thus $\ln(\omega_{max}/\omega_{min}) = (E_{max} - E_{min})/k_B T$, which for metals at low temperatures (say around 1 K) is of the order of 10^3 [29].

In the previous section it was shown that once the local persistent current changes its flow pattern from diffusive to quasi-1D, a strong suppression of $\langle \Delta I^2 \rangle / \langle I^2 \rangle$ appears. Combining these considerations, one concludes that the noise spectrum of the persistent current in the low-density regime will be suppressed. A word of caution is appropriate, since here we have considered a change in the scatterer which does not change the Wigner crystal domains in the system much. If the change in the impurity position does change the locations of the Wigner crystal domains, the change in the current might be more significant. Nevertheless, we expect that even in this case the change in the current will be local—i.e., less than in the diffusive regime.

In conclusion, we have considered some of the consequences of the transition in the local persistent-current flow pattern, due to the effects of strong electron–electron interaction, on the global properties of the persistent current. The sensitivity of the persistent current to the application of an external magnetic field, as well as to an additional impurity, is reduced in the strongly interacting regime. This will also suppress the noise autocorrelation of the persistent current, which might turn out to be a useful experimental probe into the behaviour of the local persistent current.

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